JOULE HEATING AND THERMAL DIFFUSION EFFECTS ON MHD RADIATIVE AND CONVECTIVE CASSON FLUID FLOW PAST AN OSCILLATING SEMI-INFINITE VERTICAL POROUS PLATE

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ABSTRACT

An analysis is performed to investigate the effects of Joule heating and thermal diffusion on unsteady, viscous, incompressible, electrically conducting MHD heat and mass transfer free convection Casson fluid flow past an oscillating semi-infinite vertical moving porous plate in the presence of heat source/sink and an applied transverse magnetic field. Initially it is assumed that the plate and surrounding fluid at the same temperature and concentration at all the points in stationary condition in the entire flow region. Thereafter a constant temperature is given to the plate hence the buoyancy effect is supporting the fluid to move in upward direction and is assumed that gravity is the only force which acts against to the flow direction. The governing flow is modeled in the form of partial differential equations with initial and boundary conditions. With suitable non-dimensional quantities the governing nonlinear partial differential equations obtained in dimensionless form, which are solved numerically with finite difference scheme. Numerical results for non-dimensional velocity, temperature and concentration as well as the skin-friction, the rate of heat transfer and the rate of mass transfer studied for different physical parameters. The results show that the solutal boundary layer thickness of the fluid enhances with the increase of Prandtl number and the temperature is increased by an increase in the heat source by the fluid. The central reason behind this effect is that the heat source causes an increase in the kinetic energy as well as thermal energy of the fluid. The momentum and thermal boundary layers get thinner in case of heat source fluids.

Keywords: Casson fluid, MHD, free convection, thermal diffusion, Joule dissipation, heat and mass transfer.

1. INTRODUCTION

The study of Casson fluid flow on MHD free convection flows with heat transfer past a porous plate is attracting the attention of many researchers. Casson fluid in one of such fluid, this fluid has distinct features and is quite illustrous recently. Casson fluid model was introduced by Casson in 1959 for the prediction of the flow behavior of pigment-oil suspension. So for the flow, the shear stress magnetic of Casson fluid needs to exceed the yield shear stress, or else the fluid behaves as a rigid body. This kind of fluids can be marked as a purely viscous fluid with high viscosity. Casson model is based on a structure model of the interactive behavior of solid and liquid phases of two phase suspensions. Some famous examples of Casson fluid include jelly, sauce, tomato, honey, soup and concentrated fruit juice. Human blood can also be treated as Casson fluid due to the presence of several substances such as fibrinogen, protein, globulin in aqueous base plasma and human red blood cell. In all of the above studies the solutions of Casson fluid are either obtained by using approximate method or by any numerical scheme.


Casson fluid. Bhattacharya (2009) studied boundary layer stagnation-point flow of Casson fluid and heat transfer towards a shrinking/stretching sheet. Kumar and Varma (2011) studied thermal diffusion and radiation effects on unsteady MHD flow past an impulsively started exponentially accelerated vertical plate with variable temperature and variable mass diffusion. Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation is addressed by Pramanik (2013). Mustafa et al. (2014) discussed Stagnation-point flow and heat transfer of a Casson fluid towards a stretching sheet. Recently, Rushi Kumar et al. (2015) studied thermal diffusion effects on MHD heat and mass transfer flow and surrounding fluid are at the same temperature and concentration in stationary condition for all the points in entire flow region. A uniform magnetic field of strength \( B_0 \) is applied perpendicular to the fluid flow direction.

2. MATHEMATICAL FORMULATION

In this problem, we have investigated the effects of Joule heating and thermal diffusion on unsteady MHD flow past a vertical plate in the presence of Porous Medium with uniform temperature and mass concentration under the influence of thermal radiation on unsteady heat and mass transfer MHD Casson fluid flow past a moving vertical plate when the magnetic field relative to the fluid or to the plate.

The induced magnetic field is neglected in comparison to the applied magnetic field as the magnetic Reynolds number of the flow is taken to be very small. At time \( t^* > 0 \), the plate is given an oscillatory motion with velocity \( u^* = u_0 \sin(\omega t^*) \) in its own plane. At the same time, the plate temperature is raised to \( T_w \) and the mass is diffused from the plate uniformly with mass concentration \( C_\infty \) as time advanced. The Joule dissipation is considered in the energy equation. The fluid is assumed to be gray emitting and absorbing radiation but non-scattering medium. All the fluid properties are assumed to be constant except the influence of the density variation with temperature in the body force term. Electric field and viscous dissipation effects are neglected. The constitutive equation for the Casson fluid can be written as (Mustafa et al. [2011])

\[
\tau_{ij} = \begin{cases} 
2 \left( \mu_0 + \frac{P}{\sqrt{2\pi}} \right) e_{ij}, & \pi > \pi_c, \\
2 \left( \mu_\infty + \frac{P}{\sqrt{2\pi}} \right) e_{ij}, & \pi < \pi_c,
\end{cases}
\]

Where \( \pi = e_{ij} e^*_{ij} \) and \( e_{ij} \) is the \( i, j \)th component of the deformation rate, \( \pi \) is the product of the component of deformation rate with itself, \( \pi_c \) is a critical value of this product based on the non-Newtonian model, \( \mu_B \) is plastic dynamic viscosity of the non-Newtonian fluid and \( P \) is yield stress of fluid. Under these assumptions, the equations that described the physical situation are given by

\[
\rho \frac{\partial u^*}{\partial t} - \mu_B \left( \frac{1}{\gamma} \right) \frac{\partial u^*}{\partial y} - \sigma B_0^2 u^* + \frac{\mu_\infty}{k} u^* + \rho g \beta (T - T_\infty) + \rho g \beta (C - C_\infty) \tag{1}
\]

\[
\rho C_p \frac{\partial T^*}{\partial x} = \kappa \frac{\partial^2 T^*}{\partial y^2} + Q(T - T_\infty) + \frac{\partial u^*}{\partial y} + \sigma B_0^2 u^* \tag{2}
\]

\[
\frac{\partial C^*}{\partial y} = D \frac{\partial^2 C^*}{\partial y^2} + D_1 \frac{\partial^2 T^*}{\partial y^2} \tag{3}
\]

Cogley et al. have shown that, in the optically thin limit for a non-gray gas near equilibrium, the radiative heat flux is represented by the following form:

\[
\frac{\partial q_y}{\partial y} = 4 \int L_\lambda K_\lambda \frac{\partial T^*}{\partial y} \, d\lambda
\]

The initial and boundary conditions are

\[
t^* < 0: u^* = 0, T^* = T_\infty, C^* = C_\infty \quad \text{for all } y^* < 0
\]

\[
t^* > 0: u^* = u_0 \sin(\omega t^*), T^* = T_w, C^* = C_w \quad \text{at } y^* = 0
\]

\[
u^* \to 0, T^* \to T_\infty, C^* \to C_\infty \quad \text{as } y^* \to \infty
\]

1. METHOD OF SOLUTION

Equations (5)-(7) are coupled non-linear partial differential equations and are to be solved by using the initial and boundary conditions (8). However exact solution is not possible for this set of equations and hence we solve these equations by finite-difference method. The equivalent finite difference schemes of equations for (5)-(7) are as follows:

\[
\frac{\partial u^*}{\partial t} = \frac{1}{\gamma} \frac{\partial u^*}{\partial y} - Mu^* - \frac{1}{K} u^* + Gr \theta + Gm C
\]

\[
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} + Q \theta - F \theta + M Ec u^2
\]

\[
\frac{\partial C^*}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C^*}{\partial y^2} + \frac{\partial^2 \theta}{\partial y^2}
\]

The corresponding initial and boundary conditions are:

\[
t < 0: u = 0, T = T_\infty, C = C_\infty \quad \text{for all } y < 0
\]

\[
t \geq 0: u = \sin(\omega t), \theta = 1, C = 1 \quad \text{at } y = 0
\]

\[
u \to 0, T \to T_\infty, C \to C_\infty \quad \text{as } y \to \infty
\]
\[
\frac{u_{i,j+1} - u_{i,j}}{\Delta t} = \left(1 + \frac{1}{\gamma} \right) \frac{u_{i,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta y)^2} - Mu_{i,j} - \frac{1}{K} u_{i,j} + Gr \theta_{i,j} + Gc C_{i,j} \\
\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} = \frac{1}{Pr} \left( \frac{\theta_{i,j} - 2\theta_{i,j} + \theta_{i+1,j}}{(\Delta y)^2} + Q \theta_{i,j} \\
- \frac{F \theta_{ij} + M Ec (u_{i,j})^2}{(\Delta y)^2} \right) \\
\frac{C_{i,j+1} - C_{i,j}}{\Delta t} = \frac{1}{Sc} \left( \frac{C_{i,j} - 2C_{i,j} + C_{i+1,j}}{(\Delta y)^2} + Sr \theta_{i,j} - 2\theta_{i,j} + \theta_{i+1,j} \right)
\]

Here, index \( i \) refer to \( y \) and \( j \) to time. The mesh system is divided by taking \( \Delta y = 0.04 \). From the initial condition in (8), we have the following equivalent:

\[
u(i,0) = 0, \theta(i,0) = 0, C(i,0) = 0 \quad \text{for all } i
\]

The boundary conditions from (8) are expressed in finite-difference form as follows

\[
u(0,j) = 1, \theta(0,j) = 1, C(0,j) = 1 \quad \text{for all } j
\]

\[
u(i_{\text{max}},j) = \sin(w^* (j - 1)^* \Delta t), \theta(i_{\text{max}},j) = 1, C(i_{\text{max}},j) = 1 \quad \text{for all } j
\]

(Here \( i_{\text{max}} \) was taken as 201)

First the velocity at the end of time step viz, \( u(i,j+1), (i=1,201) \) is computed from (9) in terms of velocity, temperature and concentration at points on the earlier time-step. Then \( \theta(i,j+1) \) is computed from (10) and \( C(i,j+1) \) is computed from (11). The procedure is repeated until \( t = 0.05 \) (i.e. \( j = 500 \)). During computation \( \Delta t \) was chosen as 0.0001.

**Skin-friction:**

The skin-friction in non-dimensional form is given by

\[
\tau = - \left(1 + \frac{1}{\gamma} \right) \frac{du}{dy} \bigg|_{y=0}, \text{where } \tau^* = \frac{\tau}{\rho u_0^2}
\]

**Rate of heat transfer:**

The dimensionless rate of heat transfer is given by

\[
Nu = - \left( \frac{d\theta}{dy} \right) \bigg|_{y=0}
\]

**Rate of mass transfer:**

The dimensionless rate of mass transfer is given by

\[
Sh = - \left( \frac{dC}{dy} \right) \bigg|_{y=0}
\]

### 4. RESULTS AND DISCUSSION

A Numerical study has been carried out on the MHD flow of a Casson fluid. The effects of various physical parameters such as Grashof number, Modified Grashof number, Casson parameter, Magnetic parameter, Permeability parameter, Prandtl number, Heat source, Radiation parameter, Schmidt number and Soret number on velocity, temperature and concentration are discussed with the help of graphs and Skin friction, Nusselt number and Sherwood are also studied with the help of graphs. In Fig.2, the effect of Thermal Grashof number on velocity is presented. As \( Gr \) increases, velocity also increases. This is due to the buoyancy which is acting on the fluid particles due to gravitational force that enhances the fluid velocity. A similar effect is noticed from Fig.3, in the presence of modified Grashof number, which also increases fluid velocity. Fig.4, demonstrates that the velocity decreases with an increase in Casson parameter.
force reduces the fluid velocity. Fig. 6 shows that the velocity increases with an increase in permeability parameter. This is due to the fact that increasing values of $K$ reduces the drag force which assists the fluid considerably to move faster. Fig. 7 depicts the variations in velocity profiles for different values of Schmidt number.

![Fig. 5 Effect of magnetic parameter on u](image)

![Fig. 6 Effect of permeability parameter on velocity](image)

![Fig. 7 Effect of Schmidt number on velocity](image)

From this figure it is noticed that, velocity decreases as $Sc$ increases. Physically it is true that if the concentration increases the density of the fluid increases which results a decrease in fluid particles. Fig. 8 demonstrates that the velocity decreases with an increase in Prandtl number. The velocity profile for different values of $Sr$ is plotted in the Fig. 9. It is found that the velocity increases with increasing values of $Sr$.

![Fig. 8 Effect of Prandtl number on velocity](image)

![Fig. 9 Effect of Soret number on velocity](image)

![Fig. 10 Effect of Prandtl number on temperature](image)
Fig. 10 indicates that a rise in Pr substantially reduces the temperature in the viscous fluid. It can be found from Fig.10 that the solutal boundary layer thickness of the fluid enhances with the increase of Pr. Fig.11, depicts the effect of heat source on temperature. It is noticed that the temperature is increased by an increase in the heat source by the fluid. The central reason behind this effect is that the heat source causes an increase in the kinetic energy as well as thermal energy of the fluid. The momentum and thermal boundary layers get thinner in case of heat source fluids. Fig.12 demonstrates the effect of radiation parameter on temperature. It is observed that temperature decreases as radiation parameter increases. Influence of Schmidt number on concentration is shown in Fig.13, from this figure it is noticed that concentration decreases with an increase in Schmidt number. Because, Schmidt number is a dimensionless number defined as the ratio of momentum diffusivity and mass diffusivity, and is used to characterize fluid flows in which there are simultaneous momentum and mass diffusion convection processes. Therefore concentration boundary layer decreases with an increase in Schmidt number.
Fig. 16 Effect of Casson parameter on skin friction

Pr=0.71;M=0.5;K=0.1;w=π/4;Gr=5;Sr=0.1;Sc=0.22;Ec=0.1;F=0.1;Q=0.1;γ=0.05

Fig. 17 Effect of Prandtl number on heat transfer rate

Gm=5;M=0.5;K=0.1;w=π/4;Gr=5;Sr=0.1;Sc=0.78;Ec=0.1;F=0.1;Pr=0.71;γ=0.2;Q=0.05

Fig. 18 Effect of heat source on heat transfer rate

Sc=0.22,0.60,0.78,0.96

Fig. 19 Effect of radiation parameter on heat transfer rate

F=1,5,10

Fig. 20 Effect of Schmidt number on mass transfer rate

Sh=0.22,0.60,0.78,0.96

Fig. 21 Effect of Soret number on Sherwood number

Sr=0.22,0.60,0.78,0.96
From Fig. 14, we observe that the concentration increases as Soret number increases. Figs.15 and 16 shows that skin friction decreases with increasing values of Modified Grashof Number, ‘Gm’, Casson parameter, ‘γ’, Fig. 17 we observed that the Nusselt number decreases with an increase values of Prandtl number and heat source parameter while it increase with increasing value of radiation parameter. From Fig.20 it is observed that Sherwood number increases with an increase in Schmidt number ‘Sc’ while it decreases in case of Soret Number ‘Sr’.

5. CONCLUSIONS

An analysis is performed to investigate the effects of Joule heating and Soret effect (Thermal diffusion) on unsteady, incompressible, electrically conducting radiative heat and mass transfer MHD flow of casson fluid past an infinite vertical plate with constant wall temperature and mass diffusion. The dimensionless governing equations are solved by Finite difference method. The results for velocity, temperature and concentration plotted graphically. And the following conclusions are made:

- As Gr increases, velocity also increases. This is due to the buoyancy which is acting on the fluid particles due to gravitational force that enhances the fluid velocity also it has been happen when modified Grashof number increased.
- It is found that velocity gets reduced by the increase of magnetic parameter it is due to the fact that retarding force drag the velocity while it decreases with an increase in permeability parameter.
- It is interesting note that the solutal boundary layer thickness of the fluid increases as Prandtl number increases. But a reverse effect is found with increase of radiation parameter.
- It is noticed that concentration decreases with an increase in Schmidt number however it increases an increase in soret number.
- skin friction enhances with an a reduction in Modified Grashof Number ‘Gm’ as well as Casson parameter, ‘γ’, whereas the Nusselt number decreases with an increase in Prandtl number while it increases as radiation parameter increases and finally it is observed that Sherwood number increases with an increase in Schmidt number ‘Sc’ while it decreases in case of Soret Number ‘Sr’.

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NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>( K_{in} )</td>
<td>Absorption coefficient</td>
</tr>
<tr>
<td>( N )</td>
<td>Dimensionless material parameter</td>
</tr>
<tr>
<td>( N_{lu} )</td>
<td>Local Nusselt Number</td>
</tr>
<tr>
<td>( n )</td>
<td>Scalar constant</td>
</tr>
<tr>
<td>( Pr )</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>( Q )</td>
<td>Heat source parameter</td>
</tr>
<tr>
<td>( Q_{v} )</td>
<td>Volumetric rate of heat generation or absorption</td>
</tr>
<tr>
<td>( q_w )</td>
<td>Heat flux per unit area at the plate</td>
</tr>
<tr>
<td>( R )</td>
<td>Coefficient of chemical reaction</td>
</tr>
<tr>
<td>( S_r )</td>
<td>Soret effect</td>
</tr>
<tr>
<td>( t )</td>
<td>Dimensionless time</td>
</tr>
<tr>
<td>( T )</td>
<td>Temperature in the boundary layer</td>
</tr>
<tr>
<td>( T_w )</td>
<td>Wall dimensional temperature</td>
</tr>
<tr>
<td>( T_{\infty} )</td>
<td>Ambient temperature</td>
</tr>
<tr>
<td>( u, v )</td>
<td>Components of the velocities</td>
</tr>
<tr>
<td>( x, y )</td>
<td>Co-ordinate system</td>
</tr>
</tbody>
</table>

Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>( \alpha )</td>
<td>Characteristic dimension of the flow fluid</td>
</tr>
<tr>
<td>( \beta_e )</td>
<td>Coefficient of volume expansion for heat transfer</td>
</tr>
<tr>
<td>( \beta_{e} )</td>
<td>Coefficient of volume expansion for mass transfer</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Thermal conductivity of the fluid</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>Scalar constant ((&lt;&lt;1))</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Dimensionless normal distance</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Chemical reaction parameter</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Viscosity of the fluid</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density of the fluid</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Magnetic permeability of the fluid</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Kinematic viscosity</td>
</tr>
</tbody>
</table>

Superscripts

\( ' \): Differentiation with respect to \( 'y' \)

\( * \): Dimensional parameters

Subscripts

\( w \): Wall condition

\( \infty \): Free stream condition

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