HEAT TRANSFER BETWEEN SOLID SURFACES IN CONTACT HAVING A RELATIVELY LARGE GAP (EXTERNAL THERMAL RESISTANCE OF A HEAT PIPE)

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ABSTRACT

Experimental and numerical analyses are conducted to investigate the heat transfer characteristics of two rectangular rods in contact, where a relatively large gap is present. Sixteen types of wedge-shaped gaps having different sizes are investigated in the experiment. Using the thermal resistance network method, the temperature distribution is obtained numerically and the discussion is made on the heat transfer characteristics around the gap. It is confirmed that the constriction/spreading of heat flow inside the rods greatly affects the thermal resistance of the gap. Fairly good agreement is obtained between the experimental and the numerical results.

Keywords: Electronics cooling, Thermal management, Thermal contact resistance, Wedge-shaped gap, Constriction/spreading heat flow.

1. INTRODUCTION

A heat pipe has been widely used in electronics cooling (Faghri, 2014; Mochizuki *et al.*, 2011). In this application, since the heat pipe is in contact with a heat source and a heat sink, not only internal thermal resistances of the heat pipe but also external thermal resistances, namely, thermal contact resistances of the heat pipe with the heat source and the heat sink are important factors for thermal management. Thermal interface material such as thermal grease is used to reduce the thermal contact resistance; however, it cannot be neglected if a relatively large gap is present between surfaces.

Concerning the measurement of contact heat transfer between two cylindrical specimens, the effect of curvature of contact surfaces on the accuracy of the measurement was theoretically investigated by Madhusudana (2000). Rostami *et al.* (2001) conducted the numerical analysis on the contact heat transfer between two solids. Between the contact surfaces, a gap having a shape of conical and that of circular were considered, and the thermal resistances were studied. Wahid (2003) numerically analyzed the heat flow of a cylinder having a conical gap. The numerical results were shown changing the angle of the gap and the thermal conductivity of gas in the gap. Kumar *et al.* (2004) presented a theoretical model to predict the contact heat transfer between curvilinear surfaces. Both a macro contact area corresponding to curvilinear surfaces in contact and a micro contact area for surface roughness were modeled. It was shown that the prediction agreed closely with those measured in the experiments. Bahrami *et al.* (2004) developed an approximate analytical model for determining a thermal contact resistance of nonconforming random rough contacts in the presence of an interstitial gas. Macro/micro gaps and macro/micro contacts were considered in modeling. Good agreement was shown between the calculated results by the model and the experimental data. Bahrami *et al.* (2006) also published a review

article concerning the thermal contact resistance models for nonconforming rough surfaces.

To the authors' knowledge, the number of studies concerning the contact heat transfer between solid surfaces having a relatively large gap is very small. Moreover, sufficient information has not been published on the effect of thermal grease applied to the gap. In the present study, therefore, a relatively large gap having a shape of wedge is made between solid surfaces, and the heat transfer characteristics are investigated experimentally. The thermal grease is used, and the width and the height of the wedge-shaped gap are changed. The numerical analysis is also conducted and the discussion is made on the heat transfer characteristics around the gap.

2. EXPERIMENTAL METHOD

As shown in Fig. 1, a wedge-shaped gap is made between two rectangular brass rods; the bottom surface of the upper rod is partially wedge-shaped while the top surface of the bottom rod is entirely flat. The dimensions of the rods are 25 mm × 25 mm × 60 mm (length), and the thermal conductivity is 108 W/(m·K). Sixteen types of wedge-shaped gaps categorized into two groups (G1, G2) are prepared as shown in Table 1, where a and b are the width and the height of the gap, respectively. In addition, the measurement is also conducted for the two rods in contact where the wedge-shaped gap is not made. This measurement is categorized as G0 in this study.

Figure 2 shows a schematic of a measuring apparatus. This is basically the same apparatus as used in the previous study (Tomimura *et al.*, 2011). As shown in the figure, the lower rod is placed on the cooling device, where cooling water is supplied from a thermostatic bath. The ceramic heater having the dimensions of 25 mm × 25 mm × 1 mm (thickness) is mounted on the top surface of the upper rod. A silicone grease having the thermal conductivity of 1.0 W/(m·K) is used to the gap. By using a lever and weights, a prescribed nominal contact

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Fig. 1 Wedge-shaped gap between two rods and temperature distribution (Open circles represent the measuring points of temperatures)

Table 1 Specification of wedge-shaped gaps

Gap	a [mm]	b [μ m]
G1-1	13.1	52
G1-2	13.3	108
G1-3	12.9	207
G1-4	12.8	310
G1-5	12.7	405
G1-6	12.7	610
G1-7	12.7	817
G1-8	12.9	1041
G2-1	17.7	48
G2-2	18.4	104
G2-3	18.3	211
G2-4	18.0	302
G2-5	17.6	401
G2-6	17.6	605
G2-7	17.7	820
G2-8	17.8	1027

[1] Upper rod	[4] Cooling device	[7] Weight
[2] Lower rod	[5] Thermal insulator	[8] Balance weight
[3] Ceramic heater	[6] Lever	

Fig. 2 Schematic diagram of experimental apparatus

pressure is applied to the rods. The rods are thermally insulated using a polystyrene foam. For temperature measurement, five small holes are drilled on each rod, and T-type thermocouples of 0.5 mm in diameter are inserted by applying a thermally conductive silver paste. The measuring points of the temperatures are also shown in Fig. 1.

In experiment, the top surface of the upper rod is heated by the heater while the bottom surface of the lower rod is cooled by the cooling device, and the temperature distribution at steady state is obtained by the thermocouples. The heat input and the cooling temperature are 20.5 W – 26.0 W and 10 °C – 14 °C, respectively. Although the contact surface area between the upper and the lower rod varies with the width of the gap, a , the nominal contact pressure applied on the rods is kept at 1.6 MPa throughout the experiment.

A schematic of a steady-state temperature distribution inside the rods is also shown in Fig. 1. By extrapolating the measured temperature as shown by the solid lines in the figure, the temperatures at the top surface of the upper rod, T_h , and the bottom surface of the lower rod, T_c , are calculated, and then the thermal resistance of the wedge-shaped gap, R_g , is obtained by

$$R_g = R_t - 2R_r \quad (1)$$

where R_t is the total thermal resistance between the heated and the cooled section of the rods, and R_r the thermal resistance of each rod. R_t and R_r are expressed respectively as

$$R_t = \frac{T_h - T_c}{Q} \quad (2)$$

$$R_r = \frac{L}{\lambda_r W^2} \quad (3)$$

where L , W^2 , and λ_r are the length (= 60 mm), the cross-sectional area (= 25 mm × 25 mm), and the thermal conductivity (= 108 W/(m·K)) of the rod, respectively. When one-dimensional heat flow is assumed in the rods, the heat fluxes in the upper rod, $q_{r,u}$ and the lower rod, $q_{r,l}$, are

Fig. 3 Computational domain and coordinate system

Fig. 4 Thermal resistance network for inner point [1]

calculated by using Fourier's law, and then the heat transfer rate, Q , is obtained by

$$Q = \left(\frac{q_{r,u} + q_{r,l}}{2} \right) W^2 \quad (4)$$

3. NUMERICAL ANALYSIS

The computational domain is shown in Fig. 3 with the coordinate system. The heat transfer characteristics of the upper rod, the lower rod and the wedge-shaped gap filled with the thermal grease are investigated in the x - y coordinate system. Using the thermal resistance network method, the steady-state two-dimensional heat conduction is calculated numerically under the boundary conditions of a uniform heat flux at the top surface of the upper rod (heated section), q_h , and a uniform temperature at the bottom surface of the lower rod (cooled section), T_c . The adiabatic boundary condition is exposed for the left and the right side of the computational domain.

As mentioned above, the thermal resistance network method is used to obtain the temperature distribution inside the computational domain. The followings are the representatives of discretized equations concerning the temperatures of a nodal point P indicated by [1] – [9] in Fig. 3.

For inner nodal points of the computational domain, the thermal resistance network is described as shown in Fig. 4. The temperature T_p at the inner nodal point P indicated by [1] is given by

$$\text{Point [1]: } T_p = \frac{\frac{\Delta y}{\Delta x}(T_w + T_e) + \frac{\Delta x}{\Delta y}(T_s + T_n)}{2\left(\frac{\Delta y}{\Delta x} + \frac{\Delta x}{\Delta y}\right)} \quad (5)$$

where T_w , T_e , T_s , and T_n are the temperatures at the neighboring nodal points W, E, S, and N, respectively. Δx and Δy are the x -direction and the y -direction distance between two adjacent nodal points. The derivation of this equation is described in Appendix A.

For the nodal points at the heated section, the cooled section, the left and the right side of the adiabatic section, which are indicated by [2] – [5] in Fig. 3, the temperatures T_p are given respectively by

$$\text{Point [2]: } T_p = \frac{\lambda_r \left\{ \frac{\Delta y}{2\Delta x}(T_w + T_e) + \frac{\Delta x}{\Delta y}T_s \right\} + q_h \Delta x}{\lambda_r \left(\frac{\Delta y}{\Delta x} + \frac{\Delta x}{\Delta y} \right)} \quad (6)$$

$$\text{Point [3]: } T_p = T_c \quad (7)$$

$$\text{Point [4]: } T_p = \frac{\frac{\Delta x}{2\Delta y}(T_s + T_n) + \frac{\Delta y}{\Delta x}T_e}{\frac{\Delta y}{\Delta x} + \frac{\Delta x}{\Delta y}} \quad (8)$$

$$\text{Point [5]: } T_p = \frac{\frac{\Delta x}{2\Delta y}(T_s + T_n) + \frac{\Delta y}{\Delta x}T_w}{\frac{\Delta y}{\Delta x} + \frac{\Delta x}{\Delta y}} \quad (9)$$

The following two equations are applied at the left (point [6]) and the right (point [7]) corner of the heated section:

$$\text{Point [6]: } T_p = \frac{\lambda_r \left(\frac{\Delta y}{\Delta x}T_e + \frac{\Delta x}{\Delta y}T_s \right) + q_h \Delta x}{\lambda_r \left(\frac{\Delta y}{\Delta x} + \frac{\Delta x}{\Delta y} \right)} \quad (10)$$

$$\text{Point [7]: } T_p = \frac{\lambda_r \left(\frac{\Delta y}{\Delta x}T_w + \frac{\Delta x}{\Delta y}T_s \right) + q_h \Delta x}{\lambda_r \left(\frac{\Delta y}{\Delta x} + \frac{\Delta x}{\Delta y} \right)} \quad (11)$$

The thermal resistance network for the wedge-shaped gap is shown in Fig. 5, where x_a is the x -direction distance from the left corner of the gap. The nodal point P in this network corresponds to point [8] in Fig.

Fig. 5 Thermal resistance network for interfacial point [8]

3. Inside the wedge-shaped gap, the y-direction distance between two adjacent nodal points, $\Delta y'$, is calculated by

$$\Delta y' = x_a \frac{b}{a} \quad (12)$$

It is noted here that $\Delta y'$ is not constant but increases with x_a . The temperature T_P at the interfacial point P between the upper rod and the gap indicated by [8] is given by

Point [8]:

$$T_P = \frac{\frac{\lambda_r \Delta y + \lambda_g \Delta y'}{2\Delta x} (T_W + T_E) + \frac{\lambda_g \Delta x}{\Delta y'} T_S + \frac{\lambda_r \Delta x}{\Delta y} T_N}{\lambda_r \left(\frac{\Delta y}{\Delta x} + \frac{\Delta x}{\Delta y} \right) + \lambda_g \left(\frac{\Delta y'}{\Delta x} + \frac{\Delta x}{\Delta y'} \right)} \quad (13)$$

In the same way, the temperature T_P at the interfacial point P between the lower rod and the gap indicated by [9] is given by

Point [9]:

$$T_P = \frac{\frac{\lambda_r \Delta y + \lambda_g \Delta y'}{2\Delta x} (T_W + T_E) + \frac{\lambda_r \Delta x}{\Delta y} T_S + \frac{\lambda_g \Delta x}{\Delta y'} T_N}{\lambda_r \left(\frac{\Delta y}{\Delta x} + \frac{\Delta x}{\Delta y} \right) + \lambda_g \left(\frac{\Delta y'}{\Delta x} + \frac{\Delta x}{\Delta y'} \right)} \quad (14)$$

Using an Excel spreadsheet, the calculation is conducted and the numerical result of the temperature distribution inside the computational domain is obtained.

4. RESULTS AND DISCUSSION

4.1 Comparison between Experimental and Numerical Results

Concerning the relation between the thermal resistance of the wedge-shaped gap, R_g , and the height of the gap, b , the experimental results for gap G1 and G2 are shown in Fig. 6 by symbols. The experimental result is also shown for the two rods in contact where the wedge-shaped

Fig. 6 Comparison between experimental and numerical results

gap is not made, namely, $b = 0$. This measurement is indicated by G0 in this figure. As expected, R_g increases with b because the size of the wedge-shaped gap becomes larger. The numerical results are moreover shown in this figure by lines. Because the experimental result of the thermal resistance at $b = 0$ is very small, the thermal contact resistance between the upper and the lower rod, R_c , is neglected ($R_c = 0$) and the numerical results are obtained. In spite of the neglect of R_c , fairly good agreement is found between the experimental and the numerical results, confirming the validity of the present numerical analysis.

4.2 Heat Transfer Analysis

By using the numerical model, the detailed discussion is made on the heat transfer characteristics of the wedge-shaped gap.

Heat transfer characteristics of rods and gap: The temperature distributions inside the upper rod, the lower rod, and the wedge-shaped gap are shown in Fig. 7 for (a) $b = 200 \mu\text{m}$, (b) $b = 400 \mu\text{m}$, and (c) $b = 800 \mu\text{m}$. Because the thermal conductivity of the silicone grease used in the gap ($\lambda_g = 1.0 \text{ W/(m}\cdot\text{K)}$) is much smaller than that of the rod ($\lambda_r = 108 \text{ W/(m}\cdot\text{K)}$), the constriction and the spreading of heat flow are observed near the gap inside the upper and the lower rod, respectively. Moreover, the constriction and the spreading of heat flow become conspicuous with the increase in b . The five open circles on each rod correspond to the measuring points where the thermocouples are placed in the experiment. It is found that the measuring points are not affected by the constriction and the spreading of heat flow inside the rods. Therefore, in the present experiment, the heat fluxes in the upper rod, $q_{r,u}$, and the lower rod, $q_{r,l}$, can be obtained accurately by using the measured temperature and Fourier's law under the assumption that the heat flow is one-dimensional.

When the one-dimensional heat flow in y-direction inside the upper rod, the lower rod, and the wedge-shaped gap is assumed, the thermal resistance of the gap, R'_g , is theoretically given by

$$R'_g = \left\{ \frac{\lambda_r W(W-a)}{2L} + \frac{\lambda_r \lambda_g}{\lambda_r - \lambda_g} \frac{Wa}{b} \ln \left(\frac{\lambda_r - \lambda_g}{\lambda_g} \frac{b}{2L} + 1 \right) \right\}^{-1} - 2R_r \quad (15)$$

Fig. 7 Temperature distribution inside rods and gap at $a = 12.5$ mm
(Open circles represent the measuring points of temperatures in the experiment)

where the thermal contact resistance between the upper and the lower rod, R_c , is neglected ($R_c = 0$). The derivation of this equation is described in Appendix B. It is noted here that the constriction and the spreading of heat flow inside the upper and the lower rod are not considered in Eq. (15). The numerical results, R_g , obtained by using the two-dimensional thermal resistance network method are compared in Fig. 8 with the theoretical results, R_g' , by Eq. (15). It is observed that R_g is much lower than R_g' , implying that the thermal resistance of the gap is strongly affected by the occurrence of the constriction and the spreading of heat flow inside the rods.

Effects of thermal contact resistance and contact area:

Changing the thermal contact resistance between the upper and the lower rod, R_c , the relation between the thermal resistance of the gap, R_g , and the height of the gap, b , is calculated as shown in Fig. 9. In this calculation, the thermal contact resistance, R_c , is added in the thermal resistance network between the bottom surface of the upper rod and the top surface of the lower rod. Furthermore, the relation between R_g and the width of the gap, a , is also obtained as shown in Fig. 10. In this calculation, the thermal contact resistance, R_c , is neglected ($R_c = 0$) and the numerical results are obtained for $b = 200$ μm , $b = 400$ μm , and $b = 800$ μm . The dependence of R_g on R_c and a confirms that the thermal contact resistance and the contact area of the unshaped surface are also important factors dominating the heat transfer between the surfaces in contact having the gap. This is owing to the fact that a relatively large amount of heat is transferred through the unshaped contact surfaces of the upper and the lower rod.

Effect of thermal grease: The numerical analysis is moreover conducted when the thermal grease is not used inside the wedge-shaped gap. In this case, the air having the thermal conductivity of 0.026 W/(m·K) is considered inside the gap. Concerning the relation between the thermal resistance, R_g , and the height, b , the numerical results with and without the thermal grease are shown in Fig. 11, where the thermal contact resistance, R_c , is neglected ($R_c = 0$). As expected, R_g without the thermal grease is much larger than that with the grease. For the case of gap without the thermal grease, it is found that R_g increases steeply from $b = 0$ and thereafter increases very slightly with b . For the case of gap with the thermal grease, on the other hand, R_g increases gradually from $b = 0$. The ratio of R_g with the thermal

Fig. 8 Comparison between numerical results by two-dimensional model and analytical results by Eq. (15)

grease to R_g without the grease is denoted by ξ , and also shown in Fig. 11. It is confirmed that the thermal grease is most effective for $b = 0$, namely, a flat surface without the gap; the effect of the thermal grease is reduced with the increase in b .

5. CONCLUSIONS

Experimental and numerical studies are conducted on the heat transfer characteristics of the two rods in contact having a relatively large wedge-shaped gap. The findings of this study can be summarized as follows:

- The thermal resistance network method is used in the present numerical analysis. Since the numerical results agree well

Fig. 9 Effect of thermal contact resistance on thermal resistance of wedge-shaped gap

Fig. 10 Effect of contact surface area on thermal resistance of wedge-shaped gap

with the experimental ones, the validity of the numerical analysis is confirmed.

- The constriction and the spreading of heat flow inside the upper and the lower rod are visualized. The thermal resistance of the gap is strongly affected by the occurrence of the constriction and the spreading of heat flow inside the rods.
- The thermal contact resistance and the contact area of the unshaped surface are important factors dominating the heat transfer between the surfaces in contact having the gap.
- The thermal grease is most effective for a flat surface without a relatively large gap; the effect of the thermal grease is reduced with the increase in height of the gap.

NOMENCLATURE

a width of gap (mm, m)

Fig. 11 Thermal resistance of wedge-shaped gap with and without thermal grease

b	height of gap (μm , m)
L	length of rod (mm, m)
Q	heat transfer rate (W)
q	heat flux (W/m^2)
R	thermal resistance (K/W)
R_c	thermal contact resistance (K/W)
R_g'	thermal resistance calculated by Eq. (15) (K/W)
R_t	total thermal resistance (K/W)
T	temperature ($^{\circ}\text{C}$)
W	width of rod (mm, m)
x	x coordinate (mm, m)
x_a	x -direction distance from left corner of gap (mm, m)
y	y coordinate (mm, m)

Greek Symbols

Δx	x -direction distance between adjacent nodal points (mm, m)
Δy	y -direction distance between adjacent nodal points (mm, m)
$\Delta y'$	y -direction distance between adjacent nodal points calculated by Eq. (12) (mm, m)
λ	thermal conductivity ($\text{W}/(\text{m}\cdot\text{K})$)
ξ	ratio of R_g with grease to R_g without grease (-)

Subscripts

c	cooled section
E	neighboring nodal point on east side
eff	effective
g	gap
h	heated section
l	lower
N	neighboring nodal point on north side
P	central nodal point

r	rod
S	neighboring nodal point on south side
u	upper
W	neighboring nodal point on west side

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APPENDIX A: DERIVATION OF EQ. (5)

In Fig. 4, the energy balance at the nodal point P is expressed as

$$Q_W + Q_S = Q_E + Q_N \quad (A-1)$$

where Q_W , Q_S , Q_E , Q_N are heat transfer rates at the nodal points W, S, E, N, respectively. By using Fourier's law, they are calculated respectively by

$$Q_W = \Delta y \lambda \frac{T_W - T_P}{\Delta x} \quad (A-2)$$

$$Q_S = \Delta x \lambda \frac{T_S - T_P}{\Delta y} \quad (A-3)$$

Fig. B-1 Analytical system for one-dimensional calculation

$$Q_E = \Delta y \lambda \frac{T_P - T_E}{\Delta x} \quad (A-4)$$

$$Q_N = \Delta x \lambda \frac{T_P - T_N}{\Delta y} \quad (A-5)$$

where λ is the thermal conductivity. Substitution of Eqs. (A-2) to (A-5) into Eq. (A-1) yields Eq. (5).

APPENDIX B: DERIVATION OF EQ. (15)

As shown in Fig. B-1, an element having a thickness of dx is considered. When one-dimensional heat flow in y -direction is assumed, the effective thermal conductivity of the element, λ_{eff} , is obtained from

$$\frac{2L}{\lambda_{eff}} = \frac{L - x_a b/a}{\lambda_r} + \frac{x_a b/a}{\lambda_g} + \frac{L}{\lambda_r} \quad (B-1)$$

By using λ_{eff} , the heat transfer rate from the heated to the cooled section, Q , is calculated as

$$Q = W \left(\int_0^{W-a} \lambda_r \frac{T_h - T_c}{2L} dx + \int_{W-a}^W \lambda_{eff} \frac{T_h - T_c}{2L} dx \right) \\ = \left\{ \frac{\lambda_r W (W-a)}{2L} + \frac{\lambda_r \lambda_g}{\lambda_r - \lambda_g} \frac{Wa}{b} \ln \left(\frac{\lambda_r - \lambda_g}{\lambda_g} \frac{b}{2L} + 1 \right) \right\} (T_h - T_c) \quad (B-2)$$

and therefore the total thermal resistance from the heated to the cooled section, R_t , is obtained as

$$R_t = \frac{T_h - T_c}{Q} = \left\{ \frac{\lambda_r W (W-a)}{2L} + \frac{\lambda_r \lambda_g}{\lambda_r - \lambda_g} \frac{Wa}{b} \ln \left(\frac{\lambda_r - \lambda_g}{\lambda_g} \frac{b}{2L} + 1 \right) \right\}^{-1} \quad (B-3)$$

Substitution of Eq. (B-3) into Eq. (1) yields Eq. (15).