EXAMINATION OF CONVECTIVE HEAT TRANSFER AND ENTROPY GENERATION BY TWO ADIABATIC OBSTACLES INSIDE A CAVITY AT DIFFERENT INCLINATION ANGLES

Olanrewaju M. Oyewola\textsuperscript{a,b,*} Samuel I. Afolabi\textsuperscript{b}

\textsuperscript{a} School of Mechanical Engineering, Fiji National University, Suva, Fiji
\textsuperscript{b} Department of Mechanical Engineering, University of Ibadan, Ibadan, 200213, Nigeria

\textbf{Abstract}

This paper investigates numerically the problem of convective heat transfer and entropy generation by two adiabatic obstacles positioned inside a square cavity heated at the left wall and cooled on the right wall while horizontal walls are adiabatic. The inclination angle of the cavity orientation investigated are 30, 60 and 90 degrees. Rayleigh numbers ranging from $10^3$ to $10^6$ were calculated for two vertical obstacles. The method of Galerkin finite element was employed to solve the conservation equations of mass, momentum and energy. The cavity is assumed to be filled with air with Prandtl number of 0.71. It was observed that at 30 degree inclination, temperature distribution at the top of the cavity is more pronounced compared to the temperature at the lower region where adiabatic obstacles are positioned. Consequently, increase in Rayleigh number improve this behavior in all inclination angles considered. These results show that the effects of inclination angles and adiabatic obstacles on the thermal behaviours and fluid flow characteristics within the cavity are significantly evident. Rayleigh and Nusselt numbers strongly affect heat transfer rates proportionately. Also, high temperature gradients exist at regions where entropy is generated.

\textbf{Keywords:} Square cavity, Inclination angle, Entropy generation, Adiabatic obstacles, Heat transfer.

1. Introduction

Rate of heat transfer by means of natural convection flow inside cavity have practical significance in various applications involving thermal engineering sciences as evidenced in heat exchangers, rooms’ ventilation with radiators, solar collectors, thermal storages, electronic devices cooling, lubrication and drying technologies. This is not surprising since natural convection has a strong influence and significant role to play in transportation of energy for cavities design with the goal of achieving overall heat transfer efficiency (Bejan, 1984).

The effect of size of obstacles has been explored for single obstacle in a two-dimensional square cavity by Karki et al. (2019). At the vertical hot wall, average heat transfer coefficient is directly proportional to the size of the obstacles until a higher value further leads to reduction in heat transfer rate. It was concluded that quantity of obstacles at the outer core revealed a decrease in rate of heat transfer with the nature of the obstacle being adiabatic. The effect of inclination angle in a square cavity containing a heat generating body were investigated by Das et al. (2006) as well as Nithyadevi and Umadevi (2015). Results obtained reveals heat transfer rate increases as the angle of inclination tends to zero but with increasing angle of inclination, total heat transfer in the cavity reduces drastically. Recently, Amine and Mohamed (2021) further addressed the problem of natural convection in H-form cavity with two circular adiabatic obstacles using finite element method. In this case, Rayleigh numbers represented a significant parameter for fluid flow and heat transfer behaviours. However, there were not strong effect of Rayleigh number until it reached $10^5$ when compared with results of square cavity. Moreover, Lakhal et al. (1997) investigated the natural convection in an inclined rectangular cavity with perfectly conducting fins attached on the heated wall. The effects of Rayleigh numbers, inclination angle and aspect ratio were also examined. It was observed that fins played a major role in rate of heat transfer. Low Rayleigh number was characterized by conduction dominance while inclination angle strongly affected the flow and temperature fields.

Several work on the heat transfer characteristics of various shape of solid objects inside different kind of enclosures has been receiving attentions because of its engineering application (Kim and Viskanta, 1984; Oyewola et al.2021; Oyewola et al. 2022a, 2022b). For example, the study of Kim and Viskanta (1984) revealed the outcome of solid wall on enclosure heated differentially. Several cases where heat was applied from horizontal and vertical walls of a square cavity were examined. A major conclusion that was drawn is that Nusselt number depends on parameters constituting the geometry as well as the thermal fluid flow of the walls enclosing the cavity. It worth to note that Davis Vahl de (1983) analyzed numerically natural convection in a square enclosure where the horizontal surfaces are kept adiabatic whereas the left and right surfaces are of different temperatures. This problem has been presented as the benchmark solution for a computer code validation, hence, the problem was modified to include the conjugate heat transfer task with solutions published in literatures. Investigation of House et al. (1990) have demonstrated the effect of natural convection on a conducting body at the center of an enclosure. The results of Prandtl number, Rayleigh number, thermal conductivities ratios and body size were studied.

---

\textsuperscript{*} School of Mechanical Engineering, Fiji National University, Suva, Fiji

Corresponding author: Email; ooyewola001@gmail.com
significant remark made is that heat transfer across the cavity when compared to that in the non-existence of a body, may be improved or lowered by a body whose thermal conductivity ratio is smaller or greater than one. Moreover, Sun and Amery (1997) carefully studied the influence of a heat source and an internal baffle on natural convection in a rectangular cavity. The four walls were of finite conductance. The top and bottom walls are adiabatic on the boundary while the left and right walls are heated at different temperatures. It was concluded that specifying simple boundary conditions on the walls of the cavity and neglecting the conduction through the baffles is not appropriate. Yucel and Ozdem (2003) studied natural convection in a square cavity containing partial dividers. The horizontal walls are kept adiabatic or perfectly conducting while the vertical are differentially heated. Certain disparity of the average Nusselt number with the Rayleigh number and the size of partitions were observed. Ha et al. (2002) studied two-dimensional transient natural convection within an enclosure with a square body. The bottom wall is heated and the top surface is cold. Considering the cold and hot isothermal body, neutral and adiabatic isothermal body as the boundary conditions. An unsteady flow and temperature fields was reported when the Rayleigh number is increased. They concluded that the observations depend on the thermal boundary conditions state of the body. However, the investigation of Manab and Saran (2006) focused on numerical simulation of natural convection flow in a square enclosure with given inclination angles ranging from 15° to 90° and 10\(\leq Ra\leq 10^6\) at conductivity ratios of 0.2 and 5.0 respectively. They found that critical point exists at Ra=10^3 where the mean Nusselt for low and high conductivity ratio cases swap with comparative magnitude. The analysis showed an improved heat transfer at low conductivity ratio in comparison to a body having high conductivity ratio away from the critical point.

Despite the progress made on various issues pertaining to natural convection from different view of point, there is still some unresolved issues which require more work to be done. For instance, it is observed that the study of natural convection and entropy generation by two adiabatic obstacles in an inclined cavity has not received attention (to the best of our knowledge). The knowledge of which will improve our understanding and contribute significantly to the development of engineering systems where such application is required. Therefore, this present study focus on the influence of the angle of inclination with the view to determine the effect of angle and convection parameters on the heat transfer characteristics. In order to achieve this, the various parameters considered are Ra from 10^3 to 10^6 and angle of inclination ranges from 30° to 90° for two vertical adiabatic obstacles. The ranges of these parameters are significantly important in engineering applications as discussed in the afore-mentioned literatures.

2. DESCRIPTION OF THE PROBLEM

This study considers heat transfer and entropy generation in a square cavity tilted at an inclined angle of \(\Theta=30^\circ, 60^\circ\) and \(90^\circ\) respectively.

In Figure 1, the square cavity is characterized by length \(W\), heated at the left plate and filled with a viscous fluid of Prandtl number 0.71. The heated surface is taken as Th and the right surface is taken as the cold temperature represented as Tc. Inside the cavity are two thin adiabatic obstacles fixed at the bottom wall. They are characterized by length 0.25W with a distance of 0.5W from each other.

3. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

Natural convection is governed by partial differential equations that expresses the conservation of mass, momentum and energy, this present flow considered a steady, laminar, incompressible and two-dimensional Navier-Stokes. The viscous dissipation term in the energy equation is neglected. Momentum equations in x and y-directions are simplified using Boussinesq approximation and all fluid properties are assumed to be constant except the density due to its contribution to the buoyancy force. The governing equations are thereby represented as follows:

Continuity equation

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

x- Momentum equation

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + Pr \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - Ra Pr T \cos \Theta
\]

y- Momentum equation

\[
u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + Pr \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + Ra Pr T \sin \Theta
\]

Energy equation

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

where \(u\) and \(v\) are the velocity components in the x and y directions respectively. The governing equations and the boundary conditions are transformed into dimensionless form using the following dimensionless variables.

\[
x = \frac{x^*}{W}, y = \frac{y^*}{W}, u = \frac{u^*}{a}, v = \frac{v^*}{a}, P = \frac{P^*}{\rho a^2}, \alpha = \frac{k}{\rho C_p}
\]

\[
Nu = \frac{h L}{k}, Ra = \frac{g \beta \Delta T^* W^3}{\nu a}, T = \frac{T^* - T_c}{\Delta T^*}
\]

Stream function based on the continuity equation (1) is defined as follows

\[
u = \frac{\partial \psi}{\partial y} and v = -\frac{\partial \psi}{\partial x}
\]

Heat transfer rate along the walls of the cavity was determined using a wall surface Nusselt number (Nu) which represent the ratio of convection to conduction across the boundary of the heated wall of the enclosure. The equations of local and average Nusselt number along the top wall of the cavity are stated as:

\[
Nu_{loc} = \frac{\partial T}{\partial y}
\]
\[ N_u_{avg} = \int_0^1 N_u_{loc} \, dx \]  

(7) 

In order to solve equations (1) – (4) some boundary conditions must be stated. This problem has a no-slip boundary condition on the walls, this means \( u = v = 0 \) on the two obstacles and on all four walls of the enclosure. 

At \( x = 0, \quad 0 \leq y \leq 1 \), \( T = 1 \) 

At \( x = 1, \quad 0 \leq y \leq 1 \), \( T = 0 \) 

At \( y = 0 \) and \( y = 1, \quad 0 \leq x \leq 1 \), \( \frac{\partial T}{\partial y} = 0 \) for horizontal 

Based on the above equation boundary conditions, temperature is applied to the left side of the cavity, while the right side is kept cold and the top and bottom of the cavity has no temperature gradient.

4. ENTROPY GENERATION

According to the thermodynamic equilibrium of linear transport theory, Jamal et al, (2020), the entropy generation for fluid flow is represented as 

\[ E = E_h + E_f \]  

(8) 

Where \( E_h \) = irreversibility due to heat transfer in the direction of finite temperature gradient 

\[ E_f \] = contribution of fluid friction irreversibility to the total generated entropy 

These can be further expressed in terms of 

\[ E_h = \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \]  

(9) 

\[ E_f = \phi \left[ 4 \left( \frac{\partial^2 u}{\partial x \partial y} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right] \]  

(10) 

Where \( \phi = \frac{\mu T_0 \kappa^2}{k W^2 \Delta T^2} \), is the irreversibility distribution ratio 

\[ \Delta T = T_h - T_c \]  

(11) 

5. NUMERICAL PROCEDURE AND VALIDATION OF TECHNIQUE

The governing partial differential equations are solved numerically by Galerkin finite element methods using COMSOL Multiphysics software. The non-linear partial differential equations are discretized and solved iteratively. This iterative calculus is launched by the velocity field followed by the energy equation solution and is continued until convergence is achieved. Convergence is attained by the sum of the absolute relative errors for each dependent variable in the entire flow field.

\[ \sum_{i,j} \left| \frac{\varphi_{i}^{n+1} - \varphi_{i}^{n}}{\varphi_{i}^{n}} \right| < 10^{-5} \]  

(12) 

Where \( n \) = iteration number 

\( \varphi \) = variables \( u, v \) or \( T \) 

\( i, j \) = coordinates in \( x \) and \( y \) directions

In order to ensure the validity and verification of the accuracy of numerical technique employed in this study, the results of heat transfer and entropy generation for natural convection within a square cavity heated differentially at the vertical walls with two adiabatic obstacles at the top and bottom of the cavity are compared with the results of Jamal et al, (2020) for different values of \( Ra \). The comparisons describe an excellent agreement between present results and the presented solution for different values of \( Ra \).

6. RESULTS AND DISCUSSION

Convective heat transfer and entropy generation within an inclined square cavity heated differentially at the vertical walls with two adiabatic obstacles at the bottom walls are studied numerically. The influence of changes in the inclination angles and two vertical obstacles are examined. This work is conducted for Rayleigh number, \( Ra \) ranging from \( 10^3 \) to \( 10^6 \) and Prandtl number is 0.71.

6.1 Velocity distribution

Figure 3(a)-(d) reveals the flow field in square cavity inclined at angle 30° to the horizontal, the velocity distributions in the cavity at \( Ra=10^3 \) appears stationary at the top of the cavity and at the tips of the obstacles because of no-slip boundary condition.
The intensity of the velocity reduces as Rayleigh number increase. At $Ra=10^6$, buoyancy driven force has increased in the cavity while fluid becomes more convection dominant. Moreover, cavities in Figure 4(a)-(d) were inclined at angle 60, the lowest velocity distribution is observed when $Ra=10^3$, this is more conduction dominated and the buoyancy force is very low.

Further increase in Ra number reduces the intensity of the velocity distribution and the flow field increase. In all the stages of the Ra number, normal velocity was observed at the bottom wall of the cavities, the buoyance force increases the convection mode in the cavity at high Rayleigh number.

It worth to note that velocity distribution of cavity inclined at angle 90° is examined in Figure 5(a)-(d). Importantly, at $Ra=10^5$, the fluid in the cavity experienced intensive concentration of fluid at the tips of the adiabatic obstacles and the middle-top of the cavity. This phenomenon might suggest that the flow is more conduction dominated as a result of low viscous flow within the cavity.

**Fig. 3:** Velocity profile for inclined angle 30 degree at (a)$Ra=10^3$, (b)$Ra=10^4$, (c)$Ra=10^5$ and (d)$Ra=10^6$

The effect of Rayleigh number on the temperature distribution inside the cavity with two vertical obstacles and inclination angle at 30°, 60° and 90° are depicted in Figures 6, 7 and 8 respectively.

**Fig. 4:** Velocity profile for inclined angle 60 degree at (a)$Ra=10^3$, (b)$Ra=10^4$, (c)$Ra=10^5$ and (d)$Ra=10^6$

**Fig. 5:** Velocity profile for inclined angle 90 degree at (a)$Ra=10^3$, (b)$Ra=10^4$, (c)$Ra=10^5$ and (d)$Ra=10^6$

However, the flow field gradually disappears at the introduction of higher Ra number. For instance, at $Ra=10^6$, the top of the obstacles carries a little concentration of the fluid flow and highest velocity at the hot wall. Highest velocity is obvious along the top left wall of the cavity.

**6.2 Temperature distribution**

The effect of Rayleigh number on the temperature distribution inside the cavity with two vertical obstacles and inclination angle at 30°, 60° and 90° are depicted in Figures 6, 7 and 8 respectively.

**Fig. 6:** Temperature distribution for inclined angle 30 degree at (a)$Ra=10^3$, (b)$Ra=10^4$, (c)$Ra=10^5$ and (d)$Ra=10^6$
Hot air rises at the left wall and cold air descends at the right wall in almost vertically parallel form when $Ra=10^3$, hence this phenomenon reveals a conduction phenomenon within the cavity. The hot rising air expands at the top of the cavity to fill the cold air at the right wall as the Rayleigh increases. This is due to the driven force of buoyancy caused by the change in $Ra$ number. The implication of this is that it sets the cavity into convection dominance. The presence of the obstacle at the left wall affected the rise and fall of the hot air because the obstacle separated a region of hottest air at the left wall from the coldest air at the right wall of the cavity. The effect of the inclination angle reveals that at angle $90^\circ$, highest temperature occurs at every Rayleigh number considered, this report shows the significance of orientation of cavity.

Figure 9 (a), (b) and (c) illustrates the variation of temperature distribution for inclined angle at $30^\circ$, $60^\circ$ and $90^\circ$ respectively.

Interestingly, temperature increase is obvious at the top of the cavity and lower temperature at the left wall of the cavity and the effect is being controlled by $Ra$. Hence, Rayleigh number significantly affect the buoyancy force in the cavity of consideration. Important application of this area is in heat transfer engineering which include electronic device cooling, reactor insulation etc. Figure 10 (a), (b) and (c) reveals that average Nusselt close to the left wall and at the center appears to be maximum at the cavity inclined at angle $30^\circ$ and lowest at angle $90^\circ$.

The variation of Local Nusselt in Figure 11 (a)-(c) describes the significant increase at $90^\circ$ as $Ra$ number increases. However, the higher the inclination angle, the rate of heat transfer increases within the cavity.
In regard to the flow of velocity, Figure 12 (a)-(c) shows high velocity at the top left wall and increases as the Ra number increases. However, at the middle of the cavity, lowest velocity was experienced in the cavity and reduces as the angle of inclination reduces with respect to the increase in Rayleigh numbers examined.
However, the higher the inclination angle, the rate of heat transfer increases within the cavity. In regard to the flow of velocity, Figure 12 (a)-(c) shows high velocity at the top left wall and increases as the Ra number increases. However, at the middle of the cavity, lowest velocity was experienced in the cavity and reduces as the angle of inclination reduces with respect to the increase in Rayleigh numbers examined. The keys in Figure 9-12 represent values of Rayleigh numbers.

7. CONCLUSIONS

A numerical investigation has been conducted to simulate the convective heat transfer and entropy generation by two adiabatic obstacles inside a cavity at different inclination angles. The COMSOL Multiphysics software was employed using the Galerkin finite element method to solve the incompressible, two-dimensional, laminar governing partial differential equations for the natural convection flow with mass, momentum and energy conservations. The square cavity was heated at the left vertical wall and cooled at the right wall while the top and bottom walls are insulated. Two adiabatic obstacles were positioned at the bottom of the cavity and inclined at angle 30°, 60° and 90°. With the Prandtl number of 0.71 and Rayleigh number varied from 10^3-10^6, the following results emanated:

(i) Inclination angles of the cavity strongly influence the fluid flow and thermal behaviours at 70% top of the cavity.
(ii) Rayleigh number increases the buoyancy driven force within the cavity, hence enabling conduction to convection transformation.
(iii) The presence of two adiabatic obstacles at the bottom of the cavity causes a significant influence in the thermal behavior of the fluid within the cavity. Average Nusselt number is observed to be maximum at the middle of the cavity.
(iv) Heat transfer by convection is enhanced with temperature distribution and flow characteristics influenced by the Rayleigh number parameters.
(v) For low Rayleigh numbers and low inclination angles, natural convection is suppressed and heat transfer by conduction occurred in the cavity.

Moreover, future experimental work is needed to be conducted in order to validate this simulation.

REFERENCES


https://doi.org/10.1002/fld.1650030305

https://doi.org/10.1119/cjp-2019-0504

https://doi.org/10.1016/0017-9310(96)00120-2

https://doi.org/10.1016/j.ijheatmasstransfer.2012.08.004

https://doi.org/10.1080/104077802317221393

https://doi.org/10.1016/j.ijheatmasstransfer.2006.05.041

https://doi.org/10.5937/fme2004825B

http://dx.doi.org/10.5098/hmt.17.11

https://doi.org/10.5098/hmt.18.30

https://doi.org/10.5098/hmt.18.31