NUMERICAL STUDY OF PERIODICALLY FULLY-DEVELOPED FLOW AND HEAT TRANSFER IN CHANNELS WITH PERIODIC SEMI-CIRCULAR TUBE

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\section*{ABSTRACT}

The periodically fully-developed flow and heat transfer in channels with periodic semi-circular tube is studied numerically by the direct numerical simulation (DNS), the large eddy simulation (LES), and the Reynolds stress model (RSM). When the Reynolds number is between 3000 and 25000, the Nusselt number obtained by the RSM is closer to the experimental results than the results obtained from other turbulence models. The nonlinear characteristics of flow and heat transfer is revealed based on the results of numerical simulation. When Reynolds number is high, the geometric structure and boundary conditions of the channel are symmetric, but the flow field in the channel is asymmetric. With increasing Reynolds number, the flow and heat transfer in channels experiences steady state, periodic oscillations, and finally chaos. Moreover, Nusselt number increases gradually with the decrease of tube spacing at the same Reynolds number. Among all cases considered in this paper, the maximum Nusselt number is 15.22 times of the minimum Nusselt number.

\textbf{Keywords:} Numerical simulation; Nonlinear characteristics; Turbulence model; Tube banks

\section*{1. INTRODUCTION}

Because the flue gas in the economizer of a power plant contains particulate matter such as fly ash, it will cause wear and tear to the tube banks. It is generally believed that the wear and tear is proportional to the power of 3.3 of the average flue gas velocity (Guan and Li, 2011). However, reducing wear and tear by decreasing flue gas velocity will also weaken heat transfer, which is not desirable. In order to compensate for the reduction of heat transfer due to the reduction of flue gas velocity, the tube banks are connected through fins to form a membrane economizer to enhance heat transfer. Hui \textit{et al}. (1996) compared various correlations used to calculate membrane economizers and developed more practical experimental correlations by using modeling theory to build a test bench. Song (1988) summarized the experimental correlations used to calculate membrane economizers obtained by the former Soviet scientists and proposed a simplified empirical correlation. Wang \textit{et al}. (2006) used numerical simulation to study the flow field between membrane economizers and analyzed the effect of the arrangement on heat transfer, and they found that the staggered arrangement gave higher heat transfer rates than the in-line arrangement.

Numerical simulations of other kinds of channels are also widely reported. Sabek \textit{et al}. (2016) found that the obstacles in the membrane channel could enhance the heat transfer. Sabek \textit{et al}. (2017) suggested that changing the geometry of the channel caused mixing in the flow and destroyed the boundary layer, which was also a means of heat transfer enhancement. Li \textit{et al}. (2017) studied the convective heat transfer in a channel with a triangular baffle on one side and a triangular shallow cavity on the other side; the numerical results revealed that the synergy effect was strengthened by adding baffles. Guzmán \textit{et al}. (2015) studied the convective heat transfer in wavy channels and found that the vortex enhanced flow mixing and improved the heat transfer rates. Chen \textit{et al}. (2014) studied the relationship between the convective heat transfer in the sinusoidal corrugated tubes and the geometric size. Their results showed that heat transfer was influenced by the wave height and the periodical length of corrugated tube. Hossain \textit{et al}. (2004) and Ramgadia \textit{et al}. (2013) studied flow and convective heat transfer in a sinusoidal channel numerically. Hossain \textit{et al}. (2004) found that critical Reynolds number and geometric structure affected nonlinear characteristics. Ramgadia \textit{et al}. (2013) analyzed steady flow and unsteady flow, and the results confirmed that for steady flow the heat transfer rates were very low; for unsteady flow, the increased mixing between core and near wall fluids enhanced heat transfer rates. Hafez \textit{et al}. (2011) used k-ε turbulence model to study periodically fully developed turbulent flow in a channel with a moving wavy wall, and the study showed that standard k-ε turbulence model gave better predictions than the k-ε-f\textsubscript{z} turbulence model in simulating the flow in channels of complex geometric structures. Kumar \textit{et al}. (2012) compared the heat transfer characteristics between spatial stationary and moving sinusoidal wavy wall. Abdulmassih \textit{et al}. (2016) experimentally and numerically studied the steady and unsteady mixed convection flow in a channel with the cubical open cavity and concluded that the heat transfer rates increased with the Richardson and Reynolds numbers.

For the research of convective heat transfer in crossflow over tube banks, there are often many rows of tubes, so numerical simulation incurs a high computational cost. By introducing the assumption of periodically fully developed flow and heat transfer, one could simplify the theoretical model (1988). Patankar \textit{et al}. (1977) defined the concept of periodically fully developed flow and it could simplify the tube banks

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model, save computing resources, and present the flow field and temperature field around the tube banks more clearly. Valencia et al. (2002) studied the factors affecting unsteady heat transfer characteristics in periodically fully developed channels with rectangular obstacles. Stalio et al. (2011) studied the convective heat transfer in a channel with rectangular cavities and it was found that the heat transfer strongly depended on the flow separation. Liou et al. (2002) found periodic slit ribs in a channel enhanced heat transfer significantly. Howes et al. (1997) employed a finite difference method to study the unsteady flow and came to the conclusion that the heat transfer was enhanced in a periodical obstructed channel.

Convective heat transfer in the channel will exhibit nonlinear behaviors with the change of Reynolds number and other parameters. Zhou et al. (2017) studied which turbulence model was more suitable for simulating the convective heat transfer across a single tube; the results showed that when the Reynolds numbers were low, the DNS gave good results and the LES and RSM were preferred for high Reynolds numbers. Qiu et al. (2018) simulated double-diffusive natural convection under different Prandtl numbers in the horizontal cavity and found that the mass transfer intensity increased faster than the heat transfer intensity. Fusegi (1997) simulated flow and heat transfer in a 2D channel with periodic heated cavities and found that heat transfer rates were enhanced with the increase of the Womersley number. Li et al. (2008) also adopted the assumption of periodically fully developed model to study the convective heat transfer in a 2D channel with periodic rectangular obstacles. It was found that the flow state in the channel changed from steady state to oscillatory state with the increase of the Reynolds number.

Yang et al. (2009) studied the flow and heat transfer in a grooved channel with a heated plate; they found that for high Reynolds numbers, the flow and heat transfer were self-sustained oscillating. Najam et al. (2003) studied numerically the unsteady mixed convection in a horizontal channel with a heated periodic obstacle on the wall and found that for high Reynolds numbers the heat transfer through the cold surface was reduced by the forced flow and the maximum dimensionless stream function varied with time. Korichi et al. (2007) studied the convective heat transfer in the channel with periodic mounted obstacles. The numerical results revealed that the vortex incurred disturbance which contributed to heat transfer enhancement and the oscillatory flow occurred when Reynolds numbers were high.

Yu et al. (2015) studied the effect of longitudinal and transverse pitches of the in-line tube banks on heat transfer with the numerical results; their research revealed that $N_{avg}$ increases with the increase of the pitch of the in-line tube banks for intensive array tube banks and $N_{avg}$ increases with the increase of the pitch of the in-line tube banks for sparse array tube banks. Yang et al. (2018) employed different turbulence models to simulate turbulent convection in an elliptical pipe and developed a novel formula based on the numerical results. Khan et al. (2006) found that $N_{avg}$ for tube banks in crossflow was related to Reynolds number, Prandtl number and longitudinal and transverse pitches. To the best of the authors’ knowledge, there is no report on the effect of the longitudinal and transverse pitches of tube banks on heat transfer in the channels with periodic semi-circular tube. In this paper, the nonlinear mechanism of flow and heat transfer in a channel with periodical semi-circular tube will be numerically studied and a practical numerical prediction for nonlinear engineering problems will be proposed.

2. PHYSICAL MODEL AND NUMERICAL METHODS

2.1 Physical Model

Figure 1 shows the physical model of the problem under consideration. Since the vortex cannot be fully displayed with a row of tubes, the shadow part in Fig. 1 is taken as the computational domain. The numerical simulation is then performed based on the assumption that fluid flow and heat transfer are periodically fully developed. The upper and lower walls in the flow direction are considered to be no slip stationary wall. The flow at the inlet and outlet in the flow direction are taken as the periodic boundary condition. The working fluid, flue gas, is assumed to be incompressible Newtonian fluid with a Prandtl number of 0.64, and the thermophysical properties of the working fluid are constants. The diameter of the tube is $D$, and $L_x=2.9D$, $L_y=2.15D$.

![Fig. 1 Physical model](image)

2.2 Governing Equations

The non-dimensional governing equations for the problem are:

\[
\frac{\partial U}{\partial \xi} + U \frac{\partial U}{\partial \eta} + V \frac{\partial U}{\partial \iota} = - \frac{\partial P}{\partial \eta} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial \xi^2} + \frac{\partial^2 U}{\partial \iota^2} \right) \quad (1)
\]

\[
\frac{\partial V}{\partial \xi} + U \frac{\partial V}{\partial \eta} + V \frac{\partial V}{\partial \iota} = - \frac{\partial P}{\partial \iota} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial \xi^2} + \frac{\partial^2 V}{\partial \iota^2} \right) \quad (2)
\]

\[
\frac{\partial \Theta}{\partial \xi} + U \frac{\partial \Theta}{\partial \eta} + V \frac{\partial \Theta}{\partial \iota} = \frac{1}{RePr} \left( \frac{\partial^2 \Theta}{\partial \xi^2} + \frac{\partial^2 \Theta}{\partial \iota^2} \right) \quad (3)
\]

\[
\frac{\partial \Theta}{\partial \eta} + \frac{\partial \Theta}{\partial \iota} = 0 \quad (4)
\]

where $\theta = T - T_w$, and the dimensionless variables and parameters are defined as:

\[
F = \frac{\text{Tr}_{avg}}{D}, \quad X = \frac{x}{D}, \quad Y = \frac{y}{D}, \quad U = \frac{u}{u_R},
\]

\[
V = \frac{v}{u_R}, \quad P = \frac{p}{\rho u_R^2}, \quad \Theta = \frac{T - T_w}{T_{b,s} - T_w},
\]

\[
T_{b,s} = \int_{y=0}^{L_y} \frac{1}{\rho c_p} \frac{\partial u}{\partial y} dy, \quad Pr = \frac{v}{a}, \quad Re = \frac{u_R D}{v} \quad (5)
\]

Equations (1)-(4) are subject to the following boundary conditions:

\[
Y = 0, \quad U = 0, \quad V = 0, \quad \theta = 0 \quad (6)
\]

\[
Y = L_y / D, \quad U = 0, \quad V = 0, \quad \theta = 0 \quad (7)
\]

The numerical simulation is then performed based on the assumption that fluid flow and heat transfer are periodically fully developed with the following periodic conditions:

\[
U(X, Y) = U \left( X + L_x / D, Y \right) \quad (8)
\]

\[
V(X, Y) = V \left( X + L_x / D, Y \right) \quad (9)
\]

\[
\Theta(X, Y) = \Theta \left( X + L_x / D, Y \right) \quad (10)
\]

The initial conditions are:

\[
F = 0, \quad U = 0, \quad V = 0, \quad \theta = 1 \quad (11)
\]
2.3 Numerical Methods and Assessment

2.3.1 Grid Independence Verification

In order to ensure the accuracy of the numerical simulation, it is necessary to verify grid independence. Figure 2 shows the simulation results obtained by the RSM under different grid numbers using a time-step of 0.0001s when the Reynolds number is 5000. The grid numbers of Grid1, Grid2, Grid3, Grid4, Grid5, Grid6 and Grid7 are 9, 982, 14, 361, 26, 057, 42, 821, 53, 051, 62, 665 and 97, 544. It is found that with the increase of grid number, Nu tends to be stable. The Nusselt number in periodic fully-developed region obtained by the RSM using Grid6 and a time-step of 0.00001s is 31.648. When the grid number is reduced to Grid5 and the time-step is increased to 0.0001s, the Nusselt number becomes 31.846. The relative error between these two sets of grid sizes and time-steps is less than 0.62%. Therefore, the results reported below are obtained by using the grid number of Grid5 and a time-step of 0.0001s.

![Grid independence verification.](image)

**Fig. 2** Grid independence verification.

2.3.2 Assessment of Difference Schemes

The researchers have done studied the membrane economizers and summarized the experimental correlations obtained by the former Soviet scientists. Song (1988) suggested that the experimental results of Nu can be obtained by the following correlation:

\[ Nu = 0.069 \left( \frac{L_x}{D} \right)^{-0.032} \left( \frac{L_y}{D} \right)^{-0.068} Re^{0.743} Pr^{0.44}, \]

\[ 1.45 \leq \frac{L_x}{D} \leq 3.2, 1.5 \leq \frac{L_y}{D} \leq 4.2, 3000 \leq Re \leq 25000 \]

(12)

![Comparison of different turbulence models and difference schemes predictions with experimental correlations.](image)

**Fig. 3** Comparison of different turbulence models and difference schemes predictions with experimental correlations.

Figure 3 shows the comparison of the simulation results of the second order upwind difference scheme and QUICK with experimental results. It can be seen that the simulation results of the two schemes are very close to the experimental results. Therefore, the second order upwind difference scheme is adopted in this paper.

2.3.3 Assessment of Turbulence Model

With increasing Reynolds number, the flow changes from laminar to turbulent. Different turbulence models will give different predictions. The DNS does not involve any turbulence model, but directly solving the N-S equation. Due to high-cost of computational time for the DNS, it is not practical to solve engineering problems using DNS. The LES uses the DNS to analyze the large-scale vortices and uses the RSM to analyze the small-scale vortices. The RSM solves the time-averaged N-S equation and is the most widely used approach in engineering. In Fig. 3, the DNS and the LES are used to simulate in the range of 3000 \( \leq Re \leq 25000 \) with a mesh of 53, 051 elements. The simulation results are quite different from the experimental results. When Re is 15, 000, Nu obtained by the DNS is 46.88. When Re remains unchanged and the number of grids increases to 62, 665, Nu obtained by the DNS is 47.16; when the number of grids continues to increase to 97, 544, Nu obtained by the DNS is 50.53. It can be seen that the larger the grid number is, the closer to the experimental results the DNS results are. It is believed that the DNS and the LES can simulate the flow more accurately when Re is low. When Re is high, more small vortices arise, and the small vortices can only be identified by the DNS and the LES when the grid number is sufficiently large. Compared with the instantaneous differential equation, the time averaged mathematical model used in the RSM method adding several source terms to the differential equations, which can reflect the characteristics of the vortex. Therefore, when Re is high and the grid number is small, the DNS and the LES can no longer give appropriate results. Because of the limited computational resources, in the range of Re \( \leq 3000 \), the DNS is preferred. For high Re (>3000), the RSM gives better predictions than the DNS. Therefore, the RSM is recommended for Re higher than 3000.

In this paper, the numerical simulation is carried out based on the finite volume method with the second order upwind difference scheme, and the SIMPLE method was used for the pressure and velocity coupling. The DNS and RSM models are used to simulate the flow and heat transfer for various Re with 53051 elements and a time-step of 0.0001s. These settings are based on mesh and time-step independence studies.

3. RESULTS AND DISCUSSIONS

3.1 Flow Fields for Various Re

Figure 4 shows the flow field in the channels with periodic semi-circular tube for various Re. It can be seen from Fig. 4 (a) and (d) that the flow is steady and symmetrical for Re = 50 and Re = 6000, with no interaction between the two groove circulation zones. Meanwhile, Fig. 4 (b), (c), (e), and (f) show that the flow is oscillatory with time and asymmetric vortexes appear behind the upper and the lower tubes at high Reynolds numbers. As shown in Fig. 4 (b) and (e) the streamlines begin to oscillate at the interface of the main flow and the groove circulation zone, and the shear layer is destroyed as the fluid in groove is mixed with the main flow. If the Reynolds number increases further, the streamlines become irregular and fluid mixing between the main flow and the groove fluid is greatly intensified. This is because with the increase of Re, more disturbances are formed around the tube, which cause the streamlines to oscillate. Figure 4 (e) shows the lower vortex becomes two vortices after being squeezed by the mainstream. The Reynolds number is the relative measure of inertia force and viscous force. With the increase of Re, the increased velocity in the mainstream direction leads to increased inertia force, the original large vortex is destroyed, and a smaller vortex is formed.

Because the governing equations are nonlinear partial differential equations, both symmetrical solutions and asymmetric solutions are available. With the increase of Re, even if the geometric and boundary conditions are symmetric, the real flow field is almost impossible to be symmetrical. A small disturbance will cause asymmetric vortices, and it is difficult to return to the original symmetrical state. This is a reflection of the nonlinear characteristics of the flow and heat transfer under complex geometry.
3.2 Oscillating Results

Figure 4 Flow fields for various Re: (a) Re = 50, (b) Re = 300, (c) Re = 2000, (d) Re = 6000, (e) Re = 15000, (f) Re = 80000.

Figure 5 shows variation of dimensionless temperature of a sample point with dimensionless time at \( Re = 50 \), 300 and 2000. Figure 6 shows the velocity components \( U \) and \( V \) in phase diagrams at the same sample point and Reynolds numbers. For \( Re = 50 \), the dimensionless temperature does not change with the dimensionless time, and the phase trajectory eventually tends to a point indicating that the flow and heat transfer are steady. For \( Re = 300 \), the dimensionless temperature oscillates regularly, and the phase trajectory is a closed image, which indicates that the flow and heat transfer oscillate regularly, and the phase trajectory is a closed image, which indicates that the flow and heat transfer oscillate.
periodically. For \(Re=2000\), the dimensionless temperature oscillations are intensified, and the phase trajectory is out of order which shows the flow and heat transfer are chaotic.

That variations of dimensionless temperature of the sample point (0.0928, -0.0344) with dimensionless time at \(Re=6,000\), 15,000 and 80,000 are shown in Fig. 7. The velocity components \(U\) and \(V\) in phase diagrams at the same sample point and Reynolds numbers are shown in Fig. 8. Similar to the above, the computational results show that when Reynolds number is low, the flow and heat transfer are steady and when Reynolds number is increased further, the flow and heat transfer will oscillate significantly. Therefore, both the DNS and RSM results reveal that with the increase of \(Re\), the flow and heat transfer experience steady state, periodic oscillations, and finally chaos.

Fig. 6 Velocity phase space at a sample point (a) \(Re = 50\), (b) \(Re = 300\), (c) \(Re = 2,000\).

Fig. 7 Variation of dimensionless temperature of sample point with dimensionless time (a) \(Re = 6,000\), (b) \(Re = 15,000\), (c) \(Re = 80,000\).

The time-averaged method adds a few Reynolds stress terms to the original governing differential equations, so time averaged N-S equations are nonlinear partial differential equations and the numerical solutions still oscillate. Nonlinearity is the inherent characteristics of flow and heat transfer system. As long as the relevant parameters reach the critical value, the equilibrium of the system will be destroyed.

3.3 Effects of \(L_x\) and \(L_y\) on \(Nu\)

In this paper, we set \((L_y/D \times L_x/D = 1.6 \times 1.6, 1.45 \times 1.45, 1.35 \times 1.35)\) as the intensive array tube banks, and set \((L_y/D \times L_x/D = 2 \times 3, 3 \times 3, 4 \times 3)\) as the sparse array tube banks.
Figure 9 shows the effect of $L_x$ and $L_y$ on heat transfer. In the range of $3000 < Re < 25000$, and seven different arrangement modes ($L_y/D \times L_x/D = 4 \times 3, \ 3 \times 3, \ 2 \times 3, \ 2 \times 2, \ 1.6 \times 1.6, \ 1.45 \times 1.45, \ 1.35 \times 1.35$) are set for numerical simulation.

As can be seen from Fig. 9, $Nu$ increases gradually with the decrease of $L_x$ and $L_y$ under the same $Re$. More disturbances arise because of the decrease of tube spacing, which destroys the boundary layer and enhances the heat transfer; this results $Nu$ to increase. The maximum $Nu$ appears at $Re = 25000$, $L_y/D \times L_x/D = 1.35 \times 1.35$; the minimum $Nu$ appears at $Re = 3000$, $L_y/D \times L_x/D = 4 \times 3$, and the maximum $Nu$ is 15.22 times of the minimum $Nu$.

Fig. 8 Velocity phase space at a sample point (a) $Re = 6000$, (b) $Re = 15000$, (c) $Re = 80000$.

Fig. 9 Effect of $L_x$ and $L_y$ on $Nu$.

Fig. 10 Effects of $L_y/D$ and $L_x/D$ on $Nu$ for sparse array tube banks.
As shown in Fig. 10, for three kinds of sparse array tube banks ($L_y/D \times L_x/D = 2 \times 3, 3 \times 3, 4 \times 3$), the effect of changing $L_y$ on $Nu$ is greater than that of changing $L_x$, and the effect is more obvious with higher $Re$. The increase of $L_y$ causes the mass flow rate to increase and heat transfer is enhanced.

Moreover, it is found that changing of $L_x$ has little effect on mass flow rate. The conclusion can provide a guideline to the arrangement of industrial membrane economizer.

4. CONCLUSIONS

The periodically fully-developed flow and heat transfer in channels with periodic semi-circular tube is studied numerically by the direct numerical simulation (DNS), the large eddy simulation (LES), and the Reynolds stress model (RSM). The following conclusions can be drawn from this study:

(1) When $3000 < Re < 25000$, the RSM gives better predictions than the DNS and the LES.

(2) With the increase of $Re$, the flow and heat transfer experience steady state, periodic oscillations, and finally chaos.

(3) $Nu$ increases gradually with the decrease of tube spacing at the same $Re$. For sparse array tube banks, the effect of changing $L_y$ on $Nu$ is greater than that of changing $L_x$, and the effect is more obvious with higher $Re$.

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